• Equation of the plane through $P_0 = (x_0, y_0, z_0)$ with normal vector $\mathbf{n} = (a, b, c)$:

or

or

where $d = ax_0 + by_0 + cz_0$.

• The plane through (non-collinear) points $P, Q,$ and $R$ in $\mathbb{R}^3$ has normal vector

and $d = \underline{\text{ }}$, where $P = (x_0, y_0, z_0)$.

Problem Set

1. Find the equation for the plane passing through the points $P = (5, 1, 1), Q = (1, 1, 2),$ and $R = (2, 1, 1)$.

2. Find the intersection of the line $\mathbf{r}(t) = \langle 1, 1, 0 \rangle + t \langle 0, 2, 4 \rangle$ with the plane $x + y + z = 14$.

3. Find the intersection of the plane $3x - 9y + 4z = 5$ with the $yz$-plane.

4. Find all planes in $\mathbb{R}^3$ whose intersection with the $xz$-plane is line with equation $3x + 2z = 5$. 
• Equation of the plane through \( P_0 = (x_0, y_0, z_0) \) with normal vector \( \mathbf{n} = (a, b, c) \):
\[
\mathbf{n} \cdot \langle x, y, z \rangle = d \\
\text{or} \quad a(x - x_0) + b(y - y_0) + c(z - z_0) = 0 \\
\text{or} \quad ax + by + cz = d
\]
where \( d = ax_0 + by_0 + cz_0 \).

• The plane through (non-collinear) points \( P, Q, \) and \( R \) in \( \mathbb{R}^3 \) has normal vector \( \mathbf{n} = \overrightarrow{PQ} \times \overrightarrow{PR} \) and \( d = \mathbf{n} \cdot \langle x_0, y_0, z_0 \rangle \), where \( P = (x_0, y_0, z_0) \).
1. Find the equation for the plane passing through the points $P = (5, 1, 1)$, $Q = (1, 1, 2)$, and $R = (2, 1, 1)$.

**Answer:** These points are on the same line! Hence there are infinitely many planes that pass through all three points. For instance, $y = 1$ works.

2. Find the intersection of the line $\mathbf{r}(t) = (1, 1, 0) + t(0, 2, 4)$ with the plane $x + y + z = 14$.

**Answer:** $(1, 5, 8)$.

3. Find the intersection of the plane $3x - 9y + 4z = 5$ with the $yz$-plane.

**Answer:** $-9y + 4z = 5$.

4. Find all planes in $\mathbb{R}^3$ whose intersection with the $xz$-plane is line with equation $3x + 2z = 5$.

**Answer:** $3x + by + 2z = 5$ for any $b \in \mathbb{R}$.